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Document de Travail
n°2007-10

STANDARDIZATION VERSUS PREFERENCE FOR VARIETY IN LINEAR COURNOT OLIGOPOLY

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June 2007

DT-GREQAM

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1 Introduction

The rise of modern industry, epitomized by the standardization of goods in order to reduce production costs, has led to the so-called dilemma between standardization and diversity. Consumers have a generic preference for variety, but there exist substantial costs to maintain a high level of diversity in many industries. Private producers may not be inclined to bear those costs, leading to under-provision of diversity with respect to social optimum. The issue of endogenous variety production has received recent attention, as empirical studies in international trade argue that variety promotes national (total factor) productivity, and thus growth (see for instance Feenstra and Kee [2004a, 2004b], Ardelean [2006]).

This note studies the impact of the nature of industrial competition on the tendency to under-provide variety. We investigate the point in a two-stage multi-product Cournot oligopoly model with horizontal product differentiation. In this model, consumers have a preference for variety, and we assume that when the number of goods produced by a firm increases, the cost of producing a unit of each good also increases. In stage one firms choose a number of goods to produce; in stage two they compete in quantities. We find that the tendency to under-provision is a general phenomenon, except in the polar cases of independent markets and homogenous markets.

Our paper contributes to the traditional I.O. literature on consumer preference for variety, dated at least from Chamberlin (1933), pursued thereafter in models with monopolistic competition (Dixit and Stiglitz [1977], see also d'Aspremont *et al.* [1996] and Benassy [1996]). Our main conclusion follows Meade (1974), who focuses, in the monopoly case, on the role of scale economies in the under-provision of variety (our model does not integrate scale economies of production). Interestingly, Gabszewicz (1983) shows that firms may produce less variety than the social optimum apart from any cost phenomena; this is explained by demand conditions which allow a higher extraction of the consumer surplus, the smaller the number of products. Our model also recalls Koenker and Perry (1981), who show that for many forms of imperfect competition an unregulated industry can provide either too few or too many brands as compared to the social optimum, depending on the relative strength of two opposite forces: the degree of product substitutability on one hand, and the scale economies of production on the other hand. Recently, Anderson and De Palma (2006) consider variety provision in a model of monopolistic competition with endogenous entry. The dynamics of entry typically entails too many firms

with few products *per* firm.

Our model contributes to the literature on Cournot (multi-product) oligopolies, appropriately extended to our specific technological environment (*i.e.*, one more variety increases unit production cost of all goods produced by the firm). A recent formulation of this model can be found in Sutton (1997), Symeonidis (2003a,b). We depart from these works with respect to marginal costs, assuming that firms can choose the number of their produced goods.

The paper is organized as follows. Section 2 describes formally the model, section 3 summarizes the optimal choices of firms, in terms of equilibrium quantities as well as number of goods. Section 4 focuses on welfare implications. The last section presents all proofs.

2 The model

We set up a two-stage model in which consumers have a preference for variety, while firms' unit production costs increase with the number of goods.

We consider an industry with a fixed number $N \geq 2$ of firms. Each firm produces a set of goods. We will use indices $i = 1, \dots, N$ for firms and $k = 1, \dots, n$ for products. The total number of goods is $n = \sum_{i=1}^N n_i$. We denote $\vec{n} = (n_1, \dots, n_N)$; furthermore, if $n_{-i} = (n_1, \dots, n_{i-1}, n_{i+1}, \dots, n_N)$, we may write $\vec{n} = (n_i; n_{-i})$. The technology of production entails no fixed cost. To account for the fact that the launching of a new product line generally implies high overhead costs, we assume that the marginal cost that firm i bears for each product is positively correlated to the number of goods n_i she is producing; for simplicity we assume that marginal costs are homogenous across both firms and goods. Common extra costs may also reveal intra-organizational change that affect all product lines. In a linear version, $c(n_i) = \gamma_0 + \gamma n_i$, with $\min(\gamma_0, \gamma) > 0$. In order to guarantee interior solutions we also assume $\max(\gamma_0, \gamma) < 1$. For convenience, we shall denote $\psi = \frac{1-\gamma_0}{\gamma}$.

There are S consumers, whose utility derived from consuming quantities x_1, \dots, x_n of the n goods is given by:

$$U(x) = \sum_{k=1}^n (x_k - x_k^2) - 2\sigma \sum_{k=1}^n \sum_{l < k} x_k x_l + M$$

in the region where marginal utilities (on each product) are nonnegative, *i.e.* $x_k \leq \frac{1}{2} - \sigma \sum_{l \neq k} x_l$. The consumer's inverse demand for good k is then $p_k = 1 - 2x_k - 2\sigma \sum_{l \neq k} x_l$. Firms compete in quantity, *i.e.* we will restrict ourselves to the Cournot-Nash equilibria of the game.

The timing of the game is as follows. In the first period, firms independently choose the number of goods they produce. In the second period, they compete in quantities. We search for the subgame perfect Cournot-Nash (pure strategy) Equilibria of the game.

To assess the preference for variety of consumers in this model, we select the following preference for variety parameter (see Benassy [1996]):

$$\nu(q; \sigma) = \frac{U(q, \dots, q)}{U(nq, 0, \dots, 0)}$$

This index compares the respective utility gains from concentrated *versus* distributed quantities among available goods. We obtain in our setting $\nu(q; \sigma) = 1 + \frac{(1-\sigma)(n-1)q}{1-nq}$, with the constraint that marginal utilities are nonnegative, *i.e.* $q \leq \frac{1}{2n}$ (which is the constraint on $U(nq, 0, \dots, 0)$, and noticing that the constraint on $U(q, \dots, q)$ is weaker). We can therefore state that:

Claim 1 *There is strict preference for variety for any $\sigma < 1$, with quadratic increase in q , while there is no preference for variety at $\sigma = 1$.*

3 The optimal choices of firms

We use backward induction to solve the game.

3.1 The second stage subgame

Standard computation provides the second stage equilibrium quantity of each product of firm i (Cournot quantities, profits and consumer surplus, should be denoted $f^*(\vec{n}; \sigma, \psi, N)$, $f = x_i, \pi_i, CS$; for convenience we shall abuse the notation twice: we forget parameters ψ, N and in the symmetric case we replace vector \vec{n} with integer μ). We obtain:

$$x_i^*(\vec{n}; \sigma) = \frac{1}{2(2(1-\sigma) + \sigma n_i) \left(1 + \sum_{j=1}^N \frac{\sigma n_j}{2(1-\sigma) + \sigma n_j} \right)} \\ \times \left[1 + \sum_{\substack{j=1 \\ j \neq k}}^N \frac{\sigma n_j c(n_j)}{2(1-\sigma) + \sigma n_j} - \left(1 + \sum_{\substack{j=1 \\ j \neq k}}^N \frac{\sigma n_j}{2(1-\sigma) + \sigma n_j} \right) c(n_k) \right]$$

Assuming that the equilibrium quantities of all goods owned by firm i are identical, the first order conditions indicate that the equilibrium price of product k owned by firm i is given by $p_k^*(\vec{n}; \sigma) =$

$2x_k^*(1-\sigma+\sigma n_i)+c(n_i)$. Cournot profit is therefore given by $\pi_i^*(\vec{n};\sigma) = 2S(1-\sigma+\sigma n_i)n_i \cdot x_i^*(\vec{n};\sigma)^2$.

That is,

$$\pi_i^*(\vec{n};\sigma) = \frac{S(1-\sigma+\sigma n_i)n_i}{2(2(1-\sigma)+\sigma n_i)^2 \left(1 + \sum_{j=1}^N \frac{\sigma n_j}{2(1-\sigma)+\sigma n_j}\right)^2} \\ \times \left[1 + \sum_{\substack{j=1 \\ j \neq k}}^N \frac{\sigma n_j c(n_j)}{2(1-\sigma)+\sigma n_j} - \left(1 + \sum_{\substack{j=1 \\ j \neq k}}^N \frac{\sigma n_j}{2(1-\sigma)+\sigma n_j}\right) c(n_k)\right]^2$$

This interior equilibrium is obtained under the second order conditions on quantities, positive equilibrium profits, and nonnegative marginal utilities.

3.2 The full game

The general analysis is cumbersome (starting with an arbitrary distribution of number of products \vec{n} , the number of products n_i^* maximizing firm i 's profit must be extracted from an order-6 polynomial, an order-5 one if approximating by differentiation). Focusing on symmetric equilibria, we determine Cournot quantities as $x^*(\mu;\sigma) = \frac{\gamma(\psi-\mu)}{2(2-2\sigma+\sigma(N+1)\mu)}$ under the constraint $\mu < \psi$, and Cournot profits as

$$\pi^*(\mu;\sigma) = \frac{S\gamma^2\mu(\psi-\mu)^2}{2} \cdot \frac{1-\sigma+\sigma\mu}{(2-2\sigma+\sigma(N+1)\mu)^2}$$

The condition of nonnegative marginal utilities is written as $\gamma(\psi-\mu)(1+\sigma(\mu N-1)) \leq 2(1-\sigma)+\sigma(N+1)\mu$; it can be written $\psi \leq \frac{\gamma\sigma N\mu^2 + [\gamma(1-\sigma)+\sigma(N+1)]\mu + 2(1-\sigma)}{\gamma[1+\sigma(\mu N-1)]}$. The condition holds if the RHS is greater than 1, which obtains when $\gamma < 1$.

Lemma 1 *For all $N \geq 2$, $\pi^*(\mu;\sigma)$ is either decreasing or single-peaked (increasing and then decreasing) with respect to the variable μ .*

In our analytic treatment, the equilibrium number of products *per firm* $\mu^*(\sigma)$ is approximated by continuization of μ . The expression $\mu^*(\sigma)$ is the unique positive root of an order-3 polynomial equation. The analytic solution exists (using for instance the Cardan method) but is tedious to write, so we do not present it here. We state the following proposition:

Proposition 1 *For any value of $\sigma \in]0, 1[$, the equilibrium number of products per firm decreases when the inverse measure of horizontal product differentiation increases.*

Though not straightforward to prove, this result is intuitive. A higher inverse measure of product differentiation entails more competition between firms, inducing a smaller equilibrium number of

products *per* firm. Note that the result is true for all values of ψ , that is for any productivity loss resulting from variety. Further, an immediate comparative statics indicates that the Cournot number of products *per* firm decreases with N .

To finish, the polar cases $\sigma \in \{0, 1\}$ have a unique and symmetric equilibrium in number of product *per* firm: when $\sigma = 0$, $\pi_i^*(\vec{n}; 0) = \frac{S\gamma^2}{8}n_i(\psi - n_i)^2$, and given that $n_i < \psi$ (positive equilibrium quantity), there is a unique interior solution $n_i^* = \frac{4\psi - 3 - \sqrt{4\psi^2 - 3}}{6}$. When $\sigma = 1$, $\pi_i^*(\vec{n}; 1) = \frac{S\gamma^2}{2(N+1)^2}(\psi + n - (N+1)n_i)^2$, entailing that Cournot profit is declining with n_i : there is no preference for variety when $\sigma = 1$, so producing variety only (negatively) impacts unit production costs; hence $n_i^* = 1$ for every firm i .

4 Consumer surplus and welfare analysis

This section examines the extent to which the Cournot number of products is smaller or greater than the one maximizing the social welfare. Given that the social welfare is the sum of aggregate profits and consumer surplus, it is sufficient to focus on consumer surplus, which writes as follows:

$$CS(\vec{n}, \sigma) = S \sum_{k=1}^n \left(x_k - x_k^2 - p_k x_k \right) - 2S\sigma \sum_{k=1}^n \sum_{l < k} x_k x_l$$

As for firms' profits, tractability issue forces us to focus on the symmetric number of goods μ *per* firm (the polar cases $\sigma \in \{0, 1\}$ will be treated exhaustively). Recalling the expression of Cournot quantities, we obtain the consumer surplus:

$$CS^*(\mu; \sigma) = \frac{S\gamma^2 N \mu (\psi - \mu)^2}{4} \cdot \frac{1 - \sigma + \sigma N \mu}{(2 - 2\sigma + \sigma(N+1)\mu)^2}$$

As in lemma 1, we can state that

Lemma 2 *For all $N \geq 2$, $CS^*(\mu; \sigma)$ is either decreasing or single-peaked.*

We now turn to the comparison between the Cournot number of products *per* firm and the distribution of products which maximizes consumer surplus.

Proposition 2 *For any value of $\sigma \in]0, 1[$, and considering homogenous distributions of number of goods *per* firm, the value $\tilde{\mu}$ maximizing consumer surplus is not smaller than the one maximizing firms' profits.*

The following table illustrates that firms under-provide variety with respect to social welfare, for a specific value of the cost for variety parameter $\psi = 33$ and for inverse measure of horizontal differentiation $\sigma \in \{0.05, 0.1, 0.5\}$:

N	2	3	4	5	6	10	25
$\mu^*(0.05)$	7	6	5	5	4	3	1
$\tilde{\mu}(0.05)$	8	8	8	7	7	6	5
$\mu^*(0.1)$	6	5	4	3	3	2	1
$\tilde{\mu}(0.1)$	7	7	6	6	6	5	4
$\mu^*(0.5)$	2	2	1	1	1	1	1
$\tilde{\mu}(0.5)$	3	3	3	3	2	2	1

The polar cases $\sigma \in \{0, 1\}$ represent extremal situations in which we will show that (i) there is a unique optimal distribution of products *per* firm, (ii) this distribution is homogenous and (iii) this distribution is aligned with that which maximizes consumer surplus.

When markets are independent, the Cournot consumer surplus is written

$$CS^*(\vec{n}; 0) = \frac{S\gamma^2}{16} \left(\psi^2 n + 2\psi \sum_{k=1}^n n_k + \sum_{k=1}^n n_k^2 \right)$$

As $\sum_{k=1}^n n_k = \sum_{i=1}^N n_i^2$ and $\sum_{k=1}^n n_k^2 = \sum_{i=1}^N n_i^3$, we obtain that $CS^*(\vec{n}; 0) = \frac{1}{2} \sum_{i=1}^N \pi(n_i)$.

When markets are homogenous, the Cournot consumer surplus is written after computation¹ $CS^*(\vec{n}; 1) = S \left(\frac{\gamma(N\psi - n)}{2(N+1)} \right)^2$. Denoting by $\bar{\mu}$ the average number of product *per* firm, we find that $CS(\bar{\mu} + 1; 1) - CS(\bar{\mu}; 1) \geq 0$ iff $\bar{\mu} \geq \psi - \frac{1}{2}$, which violates the positive equilibrium quantities condition $\bar{\mu} + 1 \leq \psi$. Hence, when markets are homogenous, the consumer surplus is independent of the distribution of the number of goods *per* firm and is decreasing with the total number of goods in the industry.

¹The Cournot consumer surplus is written $CS^*(\vec{n}; 1) = S((1 - p^*) \sum_{k=1}^n x_k^* - \sum_{k=1}^n x_k^{*2} - 2 \sum_{k=1}^n \sum_{l < k} x_k^* x_l^*)$, with $p^* = \frac{1+N\gamma_0+n\gamma}{N+1}$ (we notice $p_k^* = p^*$ for all products k) and $x_k^* = \frac{\gamma}{2(N+1)n_k} (\psi - (N+1)n_k + n)$. Denoting $X = \sum_{k=1}^n \sum_{l < k} x_k^* x_l^*$ and $Z = \frac{\gamma(\psi - n)}{2(N+1)}$, we obtain $X = \frac{(n-1)n\gamma^2}{8} - \frac{(n-1)NZ\gamma}{2} + Z^2 \sum_{k=1}^n \frac{1}{n_k} \sum_{l < k} \frac{1}{n_l}$. Remarking that $\sum_{k=1}^n \frac{1}{n_k} \sum_{l < k} \frac{1}{n_l} = \frac{N^2}{2} - \frac{1}{2} \sum_{i=1}^N \frac{1}{n_i}$, we can rewrite $X = \frac{(n-1)n\gamma^2}{8} + \frac{(n-1)NZ\gamma}{2} + \frac{Z^2}{2} \left(N^2 - \sum_{i=1}^N \frac{1}{n_i} \right)$. Given that $\sum_{k=1}^n \frac{1}{n_k} = N$ and that $\sum_{k=1}^n \frac{1}{n_k^2} = \sum_{i=1}^N \frac{1}{n_i}$, and replacing X by its expression, we obtain after rearrangement that $CS^*(\vec{n}; 0) = \frac{1}{2} \sum_{i=1}^N \pi(n_i)$.

5 Proofs

Proof of lemma 1. We can write $\frac{2}{\gamma^2 S} \pi^*(\mu; \sigma) = g(\mu) \cdot h(\mu; \sigma)$, with $g(\mu) = \mu(\psi - \mu)^2$ and $h(\mu; \sigma) = \frac{1-\sigma+\sigma\mu}{(2-2\sigma+\sigma(N+1)\mu)^2}$. Function g is increasing and then decreasing (given that $\mu < \psi$). We note that h is decreasing and convex. For simplicity, we will treat μ as a continuous variable. We denote for convenience $f'_x = \frac{\partial f(x)}{\partial x}$. Then, $\pi'_\mu > 0$ if and only if $\frac{g'_\mu}{g} > \frac{|h'_\mu|}{h}$. First $\pi'_\mu > 0$ requires $g'_\mu > 0$, which implies $\mu < \mu^*|_{\sigma=0}$ (when $\sigma = 0$, $h(\mu; 0)$ is constant). Second, $\frac{g'_\mu}{g} > \frac{|h'_\mu|}{h}$ if and only if

$$\frac{\psi - 3\mu}{\mu(\psi - \mu)} > \frac{\sigma \left(\sigma(N+1)\mu + 2(1-\sigma)N \right)}{(1-\sigma+\sigma\mu)(2-2\sigma+\sigma(N+1)\mu)}$$

or after development (like profit functions, we abuse the notation $y(\sigma, \psi, N)$ for $y = a, b, c, d$ for convenience, denoting $y(\sigma)$):

$$a(\sigma)\mu^3 + b(\sigma)\mu^2 + c(\sigma)\mu + d(\sigma) < 0 \quad (1)$$

with $a(\sigma) = 2\sigma^2(N+1)$, $b(\sigma) = \sigma(1-\sigma)(N+9)$, $c(\sigma) = (1-\sigma)(\psi\sigma(N-3) + 6(1-\sigma))$, $d(\sigma) = -2\psi(1-\sigma)^2$. Equivalently, we may write

$$a(\sigma)\mu^2 + b(\sigma)\mu + c(\sigma) < \frac{-d(\sigma)}{\mu} \quad (2)$$

Since $a(\sigma) > 0$, $b(\sigma) > 0$ and $d(\sigma) < 0$, the LHS is positive and increasing while the RHS is positive and decreasing: there is a unique intersection at a positive number; this number is $\mu^*(\sigma)$. Therefore, when $\mu > 1$, profit is either decreasing or single-peaked in μ . \square

Proof of proposition 1. We divide the proof in five parts: (i) $N = 2$ when $\sigma < \frac{1}{2}$, (ii) $N = 2$ when $\sigma > \frac{1}{2}$, $N = 3$ and $N \geq 4$ when $\psi < \frac{3N-5}{N-3}$, (iii) $N \geq 4$ when $\sigma < \frac{1}{2}$, (iv) for all $N \geq 5$ when $\sigma > \frac{1}{2}$ and $N = 4$ when $\sigma > 0.6$, (v) $N = 4$ when $\sigma \in [0.5, 0.6]$,

(i). Assume $N = 2, \sigma < \frac{1}{2}$. Fix some admissible ψ and μ . Then, $\pi(\mu) > \pi(\mu+1)$ iff

$$[1 - X(\sigma; \mu)]\psi^2 - 2[(1 - X(\sigma; \mu))\mu + 1]\psi + (\mu+1)^2 - X(\sigma; \mu)\mu^2 < 0 \quad (3)$$

with $X(\sigma; \mu) = \frac{\mu}{\mu+1} \frac{(1-\sigma+\sigma\mu)(2+\sigma+3\sigma\mu)^2}{(1+\sigma\mu)(2-2\sigma+3\sigma\mu)^2}$. We check that for all μ , all $\sigma \leq 0.5$, both $X(\sigma; \mu) < 1$ and $\frac{\partial X(\sigma; \mu)}{\partial \sigma} > 0^2$. The greatest root of equation (3) is written $\psi''(\sigma; \mu) = \frac{\mu+1-X(\sigma; \mu)\mu + \sqrt{X(\sigma; \mu)}}{1-X(\sigma; \mu)}$.

² $X(\sigma; \mu) < 1$: indeed, we obtain after little computation that $X(\sigma; \mu) < 1$ iff $(3\mu^2 - 13\mu + 4)\sigma^2 + (8\mu - 8)\sigma + 4 >$

We find that this root is increasing in σ : $\frac{\partial \psi''(\sigma; \mu)}{\partial \sigma} > 0$ iff $\frac{\partial X(\sigma; \mu)}{\partial \sigma} \cdot (2 + \sqrt{X(\sigma; \mu)} - X(\sigma; \mu))$. Since $X(\sigma; \mu) < 1$, we obtain that the sign of $\frac{\partial \psi''(\sigma; \mu)}{\partial \sigma} > 0$ is that of $\frac{\partial X(\sigma; \mu)}{\partial \sigma} > 0$, and thus it is positive.

Furthermore, the smallest root is written $\psi'(\sigma; \mu) = \frac{\mu+1-X(\sigma; \mu)\mu-\sqrt{X(\sigma; \mu)}}{1-X(\sigma; \mu)}$. We find that this root is decreasing in σ : indeed, $\frac{\partial \psi'(\sigma; \mu)}{\partial \sigma} > 0$ iff $\frac{\partial X(\sigma; \mu)}{\partial \sigma} \cdot (2 - X(\sigma; \mu) - \sqrt{X(\sigma; \mu)}) > 0$. Since $X(\sigma; \mu) < 0$, we obtain that the sign of $\frac{\partial \psi'(\sigma; \mu)}{\partial \sigma} > 0$ is the opposite of that of $\frac{\partial X(\sigma; \mu)}{\partial \sigma}$.

Then we are done: if for some σ_0 we have $\pi(\mu) > \pi(\mu + 1)$, then it is the case for any $\sigma_1 > \sigma_0$ (see figure 1).

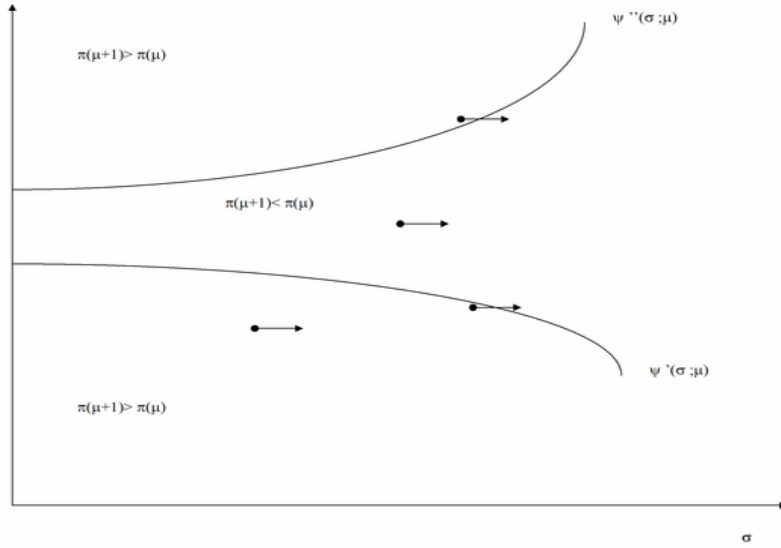


Figure 1: Separation *loci* of $\pi(\mu), \pi(\mu + 1)$; X-axis: σ - Y-axis: $\psi'(\sigma; \mu), \psi''(\sigma; \mu)$

(ii). Given that the RHS of inequality (2) is decreasing in σ , a sufficient condition is that

$(3\mu^2 - 5\mu)\sigma^3$. This last inequality is implied by $(3\mu^2 - 13\mu + 4)\sigma^3 + (8\mu - 8)\sigma + 4 > (3\mu^2 - 5\mu)\sigma^3$, or equivalently $(8\mu - 4)\sigma(1 - \sigma^2) + 4(1 - \sigma) > 0$, which is true. Further, $\frac{\partial X(\sigma; \mu)}{\partial \sigma} > 0$: indeed, since $X(., .) > 0$, we can study the sign of its logarithmic derivative, which implies $\frac{\partial X(\sigma; \mu)}{\partial \sigma} > 0$ iff $2 \left[\frac{3\mu+1}{2+\sigma(3\mu+1)} - \frac{3\mu-2}{2+\sigma(3\mu-2)} \right] > \frac{\mu}{1+\sigma\mu} - \frac{\mu-1}{1+\sigma(\mu-1)}$, with both RHS and LHS being positive quantities. This inequality reduces as $3\sigma^2\mu^2 + 3\sigma(4-3\sigma)\mu + 2(1-\sigma)(4-\sigma) > 0$. Expressed as a polynomial in μ , the greatest root exists for all σ and it is easily seen to be negative, implying that the derivative is positive.

the LHS is increasing. Now, the derivative of the LHS is written $a'_\sigma(\sigma)\mu^2 + b'_\sigma(\sigma)\mu + c'_\sigma(\sigma)$ and $a'_\sigma(\sigma) > 0$ for all $\sigma > 0$ (denoting for convenience $y'_\sigma = \frac{\partial y(\sigma)}{\partial \sigma}$, $y = a, b, c$). When the discriminant of this order-2 polynomial expression in μ is negative, we are done; otherwise, we notice that the smallest root is negative, while the greatest root is smaller than 1 whenever

$$\sqrt{(1-2\sigma)^2(N+9)^2 - 16(N+1)\sigma[(1-2\sigma)(N-3)\psi - 12(1-\sigma)]} < 8\sigma(N+1) + (1-2\sigma)(N+9)$$

i.e.

$$\sigma[N-1-\psi(N-3)] > -\frac{1}{2}(N-3)(1+\psi) \quad (4)$$

First note that the condition is valid when $N = 3$ or when $N = 2$ if when $\sigma > \frac{1}{2}$. Second, for $N \geq 4$, if $\psi < \frac{N-1}{N-3}$, we are done; if $\psi \geq \frac{N-1}{N-3}$, equation (4) is written $\sigma < \frac{(N-3)(1+\psi)}{2[\psi(N-3)-(N-1)]}$, whose RHS is greater than 1 as soon as $\psi < \frac{3N-5}{N-3}$.

(iii). Suppose that $\mu^*(\sigma)$ is locally increasing. This means that the derivative of the order-3 polynomial expression given in equation (1) is locally negative around $\mu^*(\sigma)$. Noticing that $a'_\sigma = \frac{2}{\sigma}a$, $b'_\sigma = b \cdot \frac{1-2\sigma}{\sigma(1-\sigma)}$, $c'_\sigma = c - 6(1-\sigma)(3-\sigma) + \psi(N-3)(\sigma^2 - 3\sigma + 1)$ and $d'_\sigma = -\frac{2d}{1-\sigma}$, we obtain that $a'_\sigma\mu^3 + b'_\sigma\mu^2 + c'_\sigma\mu + d'_\sigma < 0$ iff

$$\mu > \frac{1}{(1-\sigma)(3-\sigma)} \left[\frac{a\mu^3}{3\sigma} + \frac{(1-2\sigma)b\mu^2}{6\sigma(1-\sigma)} + \frac{(c + \psi(N-3)(\sigma^2 - 3\sigma + 1))\mu}{6} - \frac{d}{3(1-\sigma)^2} \right] \quad (5)$$

As $\mu^3 \geq \mu^2 \geq \mu$, when $\sigma \leq \frac{1}{2}$ inequality (5) implies $\mu > Y \cdot \mu - \frac{d}{3(1-\sigma)^3(3-\sigma)}$ with

$$Y = \frac{2(1-\sigma)a + (1-2\sigma)b + \sigma(1-\sigma)c + \sigma(1-\sigma)(\sigma^2 - 3\sigma + 1)\psi(N-3)}{6\sigma(1-\sigma)^2(3-\sigma)}$$

As $d < 0$, inequality (5) is therefore invalid if $Y \geq 1$, which means

$$4\sigma(N+1) + (1-2\sigma)(N+9) + (1-\sigma)[6 + \sigma(\psi(N-3) - 6)] + (N-3)\psi(\sigma^2 - 3\sigma + 1) \geq 6(1-\sigma)(3-\sigma)$$

i.e. $N-3 + 2\sigma(N-1)\psi(N-3)(1-2\sigma) \geq 0$. This inequality holds when $\sigma < \frac{1}{2}$.

(iv). We show that $\pi(1;\sigma) > \pi(2;\sigma)$ when $\psi > 2$. Since we know from the point (ii) that the case $\psi < \frac{3N-5}{N-3}$ is covered and that $\frac{3N-5}{N-3} > 2$ for all $N > 0$, we will be done.

The inequality $\pi(1;\sigma) > \pi(2;\sigma)$ writes:

$$\frac{(\psi-1)^2}{[2-2\sigma+\sigma(N+1)]^2} > \frac{2(1+\sigma)(\psi-2)^2}{[2-2\sigma+2\sigma(N+1)]^2}$$

that is, $(X-1)\psi^2 - 2(X-2)\psi + X - 4 > 0$, with $X = \frac{2}{1+\sigma} \cdot \left(\frac{1+\sigma N}{2+\sigma(N-1)}\right)^2$. The associated reduced discriminant is positive and equal to X . We note that $X < 4$: indeed, $X < 4$ iff

$7 + 6N\sigma + (N^2 + 4N - 6)\sigma^2 + 2(N - 1)^2\sigma^3 > 0$, which is true for any $N > 1$. We also note that $X > 1$ for all $N \geq 5$ and for $N = 4$ if $\sigma \geq 0.6$: indeed, $X > 1$ iff $N^2 - 2N + 1 - \frac{2(1+\sigma)}{\sigma^2} > 0$. Note that $\Delta' = \frac{2(1+\sigma)}{\sigma^2} > 0$, and $N'' = 1 + \frac{\sqrt{2(1+\sigma)}}{\sigma}$. We check that $N'' < 5$ for all $\sigma \in [0.5, 1]$, and for $N'' < 4$ when $\sigma < 0.6$ ($N''(\sigma)$ is decreasing in the interval, from 4.46 to 3).

In the region where $X > 1$, obtaining $\pi(1; \sigma) > \pi(2; \sigma)$ requires $\psi > \psi''$, with $\psi'' = \frac{X-2+\sqrt{X}}{X-1}$ ($\psi' < 0$ as $X < 4$). To finish, we remark that $\psi'' < 2$ iff $X > 1$.

(v). We obtain after little development that $\pi(1; \sigma) \geq \pi(2; \sigma)$ iff $\psi \leq \psi_c(\sigma)$, with $\psi_c(\cdot)$ increasing in $[0.5, 0.6]$ and $\psi_c(0.5) \simeq 98.49$. So we are done if $\psi \leq 98$.

Furthermore, $\pi(3; \sigma) \leq \pi(2; \sigma)$ iff $(B-1)\psi^2 - 2(2B-3)\psi + 4B-9 \geq 0$, with $B = \frac{1}{6} \left(\frac{1+\sigma}{1+2\sigma} \right) \left(\frac{2+13\sigma}{1+4\sigma} \right)^2$.

We notice that both $B - 1 > 0$ and $4B - 9 < 0$ for $\sigma \in [0.5, 0.6]$. Then we need $\psi \geq \frac{2B-3+\sqrt{(2B-3)^2-(B-1)(4B-9)}}{B-1}$. A straightforward inspection indicates that this latter RHS is smaller than 3 for all admissible values of σ . Then, for $\psi > 98$, $\mu^*(\sigma) \leq 2$ in $[0.5, 0.6]$.

The result follows. Let us assume some exogenous value ψ . As $\psi_c(\cdot)$ is increasing in σ , if $\pi(2; \sigma_0) < \pi(1; \sigma_0)$ for a given value σ_0 in $[0.5, 0.6]$, then for any $\sigma_1 > \sigma_0$, $\pi(2; \sigma_1) < \pi(1; \sigma_1)$; and if $\pi(2; \sigma_0) > \pi(1; \sigma_0)$ for a given value σ_0 in $[0.5, 0.6]$, then for any $\sigma_1 > \sigma_0$, it cannot be the case that $\pi(2; \sigma_1) < \pi(3; \sigma_1)$. ■

Proof of lemma 2. The proof is similar to that of lemma 1. Replacing adequately function $h(\mu; \sigma)$ with function $l(\mu; \sigma) = \frac{1-\sigma+\sigma N\mu}{(2-2\sigma+\sigma(N+1)\mu)^2}$, we notice that l is decreasing and convex, and $\frac{g'_\mu}{g} > \frac{|l'_\mu|}{l}$ if and only if

$$\frac{\psi - 3\mu}{\mu(\psi - \mu)} > \sigma \frac{2 - 2\sigma + \sigma N(N+1)\mu}{(1 - \sigma + \sigma N\mu)(2 - 2\sigma + \sigma(N+1)\mu)}$$

or after development

$$A(\sigma)\mu^2 + B(\sigma)\mu + C(\sigma) < \frac{-D(\sigma)}{\mu} \quad (6)$$

with $A(\sigma) = 2\sigma^2 N(N+1)$, $B(\sigma) = \sigma(1-\sigma)(9N+1)$, $C(\sigma) = (1-\sigma)[6(1-\sigma) - \psi\sigma(3N-1)]$, $D(\sigma) = -2\psi(1-\sigma)^2$. We have $A(\sigma) > 0$, $B(\sigma) > 0$, $D(\sigma) < 0$, so the LHS in inequality (6) is increasing while the RHS is decreasing; that is, the two curves have a unique intersection when $\mu > 0$. □

Proof of proposition 4. The Cournot consumer surplus can be written

$$CS^*(\mu; \sigma) = \frac{N}{2} \left(\frac{1-\sigma+\sigma N\mu}{1-\sigma+\sigma\mu} \right) \pi^*(\mu; \sigma)$$

Then,

$$\frac{\partial CS^*(\mu; \sigma)}{\partial \mu} = \frac{\partial \pi^*(\mu; \sigma)}{\partial \mu} \left(\frac{1 - \sigma + \sigma N \mu}{1 - \sigma + \sigma \mu} \right) + \pi^*(\mu; \sigma) \left(\frac{\sigma(1 - \sigma)(N - 1)}{(1 - \sigma + \sigma \mu)^2} \right)$$

We conclude that if $\frac{\partial \pi^*(\mu; \sigma)}{\partial \mu} > 0$, then $\frac{\partial CS^*(\mu; \sigma)}{\partial \mu} > 0$. Recalling by lemma 1 that $\pi^*(\mu; \sigma)$ is single-peaked, the result follows. ■

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